Influence of caudal fin elasticity on swimmer propulsion

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Abstract The aim of this study is to evaluate the influence of caudal fin elasticity on swimmer propulsion. The swimmer paradigm is a simplified fish model where the fins are limited to a caudal one. This caudal fin can be either rigid or elastic. The fin spine elasticity is modeled by lumped springs and dampers. The effect of tail rigidity will be illustrated on 3D self propelled fishes.

1 Introduction

The modeling and simulation of fish-like swimming is of interest in life sciences as well as in engineering applications. Understanding the mechanics of swimming can help in clarifying some aspects of the biological evolution and of the physiology of aquatic organisms. In engineering the study and optimization of aquatic locomotion can improve the design of underwater vehicles having superior maneuvering capabilities. Efficiency is also an important point, for example to minimize power or to maximize cruise velocity. It can be observed that some fishes have elastic caudal tails and we plan to study the influence of elastic caudal tails on efficiency. Therefore the scope of the paper is twofold. We present a simulation technique that is an extension to moving objects of the method explained in [1] and investigate the influence of caudal fin elasticity on swimmer propulsion.

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2 Modeling of 3D swimmers with elastic caudal tail

We considered a rigid fish of unit length, $\ell = 1$. At rest, the midline (backbone) of the fish coincides with $0 \leq x \leq 1$, $y = z = 0$. This backbone is then discretized into vertebrae located at x_i for $i = 1,..,N$. The 3D fish shape is composed with $N = 256$ ellipses with axis $y(x_i)$ and $z(x_i)$. The $y(x)$ the $z(x)$ axis are parametrized using B-splines (see figure 1). The maximum transverse dimensions are $2y = 0.17$ and $2z = 0.24$. The caudal fin has maximal vertical span $2z = 0.25$.

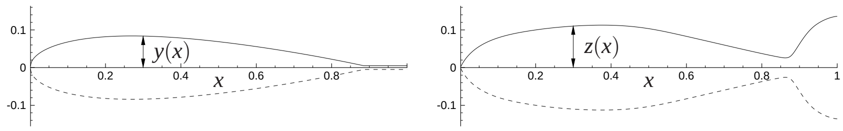


Fig. 1 Representation of the ellipse axes $y(x)$ and $z(x)$ defining the fish shape.

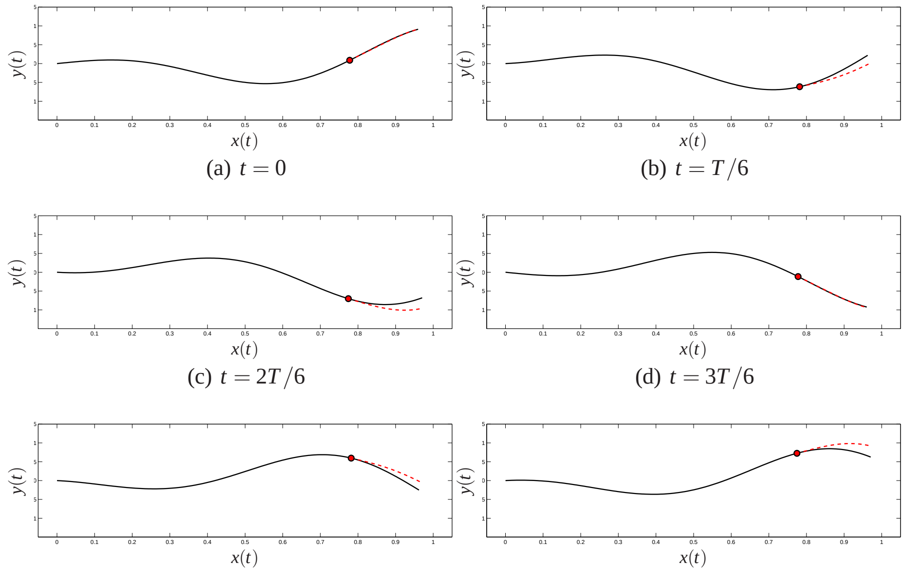
The swimming shape is obtained by deforming the midline in a plane $z = 0$. Each ellipse remains orthogonal to the midline during deformation. The whole body surface, noted $\partial\Omega_s$, can be discretized as follows $\partial\Omega_s = \cup_{i=1}^{N_s} \partial\Omega_s^i$, where N_s denotes the number of surface sections. The velocity of the body surface, noted $\boldsymbol{u}_s(\boldsymbol{x}_i, t)$, is then computed tracking the Lagrangian markers for $i = 1,..N_s$. The midline is usually deformed using the sinusoidal law [3, 2]:

$$y(x) = (c_1 x + c_2 x^2) \sin(kx - \omega t), \tag{1}$$

where $k = 2\pi/\lambda$ denotes the wave number associated with a wavelength λ and $\omega = 2\pi f$ denotes the pulsation of the oscillations associated with frequency f. The amplitude envelope $c_1 x + c_2 x^2$ is defined by two parameters c_1 and c_2 that can be adjusted to reach a desired maximal tail excursion, A. The backbone length remains $\ell = 1$.

This deformation can be either imposed on the entire or on a part of the midline. The explicitly imposed deformation of the whole midline corresponds to the solid lines in figure 2 that represent the deformation over one swimming stroke T.

The deformation can also be imposed on the midline excluding the caudal tail where an elastic medium is considered (dotted red line in figure 2). The motion of the elastic caudal tail, modeled by a series of struts with links connected by a junction (the red circle in figure 2), is computed as spring/damper system. The forcing term is proportional to the speed of each link. The variables are the rotation angle of each link with respect the previous one. The system of ODEs is derived from Hamiltonian mechanics. This ODEs system is then coupled with a flow model. It can be seen in figure 2 that the total excursion of the elastic tail is higher than the rigid one. It results in a higher maximum stroke velocity at the tail extremity near $y = 0$.



(a) $t = 0$ (b) $t = T/6$

(c) $t = 2T/6$ (d) $t = 3T/6$

(e) $t = 4T/6$ (f) $t = 5T/6$

Fig. 2 Deformation of the midline (backbone) over one swimming stroke T. —— the whole body (black), − − − elastic tail with junction o (red).

3 Flow modeling and numerical method

The modeling of flows past deformable bodies and the numerical methods are basically the same than those described in [5] and [3]. The flow including the swimmer is modeled with the penalized Navier-Stokes equation discretized on a Cartesian mesh. The domain under consideration is a 3D box $\Omega = \Omega_f \cup \Omega_s$ (the "aquarium"), where Ω_f is the domain filled by fluid, and Ω_s is the domain defined by the swimmer. The penalized incompressible Navier-Stokes equations [1] are:

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right) = -\nabla p + \mu\Delta\boldsymbol{u} + \frac{1}{K}\chi(\overline{\boldsymbol{u}} - \boldsymbol{u}) \text{ in } \Omega, \tag{2a}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \text{ in } \Omega, \tag{2b}$$

with initial conditions $\boldsymbol{u}(\boldsymbol{x},0) = \boldsymbol{u}_0(\boldsymbol{x})$ in Ω_f, and boundary conditions $\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{0}$ on the box boundaries. The body is taken into account by the penalty term $\frac{1}{K}\chi(\overline{\boldsymbol{u}} - \boldsymbol{u})$. It is modeled as a porous medium with very low permeability K. The characteristic function is $\chi = 1$ where $\boldsymbol{x} \in \Omega_s$ and $\chi = 0$ elsewhere. The first order penalized velocity $\overline{\boldsymbol{u}}$ is usually imposed [1] and results in $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}} = \boldsymbol{0}$ inside the body ($\boldsymbol{n}$ is the outward normal to the body), so that $\overline{\boldsymbol{u}} = \boldsymbol{u}_s$ on the first nodes inside the body. A second order penalization can easily be obtained using a discrete forcing version of the immersed boundary method [6, 5]. In this case, a second order approximation $\boldsymbol{u}$ of $\boldsymbol{u}_s$ is imposed considering that $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}}$ is continuous across the interface (the sharp interface).

Equations (2) are discretized in time using a predictor-corrector scheme [4] and spatially discretized on a fixed Cartesian mesh with with a second/third order method with $\Delta \boldsymbol{x} = (\Delta x, \Delta y, \Delta z)$.

Formally, the body velocity can be written as:

$$\boldsymbol{u}_s = \overline{\boldsymbol{u}} + \widehat{\boldsymbol{u}} + \widetilde{\boldsymbol{u}}. \tag{3}$$

where $\widetilde{\boldsymbol{u}}$ is the deformation velocity, $\overline{\boldsymbol{u}}$ is the translation and rotation velocity and $\widehat{\boldsymbol{u}}$ is the rigid rotation velocity. While the deformation velocity is imposed for the swimming, the translation and rotation velocity are computed from forces and torques using the Newton laws. The force exerted by the fluid on the body surface $\partial \Omega_s^i$ is $\boldsymbol{F}^i = (F_x^i, F_y^i, F_z^i)^T = -\int_{\partial \Omega_s^i} (-p\boldsymbol{I} + \frac{1}{Re}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)) \cdot \boldsymbol{n} \, \mathrm{d}s$, where $\boldsymbol{n}$ is the unit outward vector to $\partial \Omega_s^i$. The total work done by the swimmer over one stroke T is then:

$$W_{total} = \int_T \sum_{i=1}^{N} \boldsymbol{F}^i \cdot \boldsymbol{u}^i \, \mathrm{d}t, \tag{4}$$

where $\boldsymbol{u}^i$ is the average velocity on the surface $\partial \Omega_s^i$. The useful work is defined as the part of the total work where the fluid exerts a force in the swimming direction [7]. For instance, if the fish velocity is positive in the x-direction, $U_x > 0$, the useful force is $(F_x + |F_x|)/2$. The useful work is thus:

$$W_{useful} = \int_T \sum_{i=1}^{N} \frac{F_x^i + |F_x^i|}{2} U_x^i \, \mathrm{d}t. \tag{5}$$

Since the boundary $\partial \Omega_s$ does not fit the fluid mesh, $\boldsymbol{u}$ and p are obtained thanks to Lagrange interpolation. The propulsive efficiency can be defined as

$$\eta = \frac{W_{useful}}{W_{total}},$$

where W_{total} and W_{useful} are total and useful work done by the swimmer over one stroke.

4 Numerical results

This section is devoted to present our first results obtained with an elastic caudal tail. Our results correspond to low Reynolds numbers ($Re = 10^3$) relative for instance to a small fish $3\,cm$ long, swimming at velocity around one body length per second. The frequency is $f = 2$, the wavelength is $\lambda = 1$ and c_1 is chosen to obtain $A = 0.1$

($c_2 = 0$). The tail elasticity parameters (mass/spring/damping) are chosen to obtain the same deformations as in figure 2.

Results are summarized in table 1. The main conclusion is that the elastic tail allows to increase the average velocity and the efficiency. However, some elasticity coefficients still have to be optimized to maximize the efficiency.

Table 1 Comparison of average swimming velocity and spent energy with or without elastic tail. The percentages represent the augmentations with respect to values obtained with ”rigid” tail.

Re	elastic tail lumped parameter	V	η
10^3	imposed deformation	0.640	0.285
	rigid elasticity, 4.10^{-2}	0.734	0.261
	mid elasticity, 8.10^{-2}	0.728	0.345
	soft elasticity, 16.10^{-2}	0.422	0.256

5 Conclusions

We have modeled the caudal fin elasticity of a swimmer by lumped parameters. Our preliminary results show that tail elasticity has an influence on swimmer propulsion. The useful effect is that the swimming velocity is increased. The next step is to compare the efficiency for same swimming velocities. Same velocity can be imposed with a regulator on the swimming law (amplitude and frequency).

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